

Pulsatile flow and heat transfer of a magneto-micropolar fluid through a stenosed artery under the influence of body acceleration

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Abstract

With an aim to investigate the effect of externally imposed body acceleration and magnetic field on pulsatile flow of blood through an arterial segment having stenosis is under consideration in this paper. The flow of blood is presented by a unsteady micropolar fluid and the heat transfer characteristics have been taken into account. The non-linear equations that governing the flow are solved numerically using finite difference technique by employing a suitable coordinate transformation. The numerical results have been observed for axial and microrotation component of velocity, fluid acceleration, wall shear stress(WSS), flow resistance, temperature and the volumetric flow rate. It thus turns out that the rate of heat transfer increases with the increase of Hartmann number H , while the wall shear stress has a reducing effect on the Hartmann number H and an enhancing effect on microrotation parameter K as well as the constriction height δ .

Keywords: Stenosis, Micropolar Fluid, Magnetic field, Body acceleration.

1 Introduction:

It is well known that fluid dynamical behaviour of blood through an arterial segment having stenosis play a vital role in cardiovascular disease. The narrowing in the artery, commonly referred to as stenosis, is a dangerous disease and is caused due to the abnormal growth in the

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lumen of the arterial wall. Stenosis may be formed at one or more locations of the cardiovascular system. As a result of such undesirable formation at the endothelium of the vessel wall, reduction of regular blood flow is likely to take place near the stenosis. If this disease takes a severe form, it may lead to stroke, heart attack and various cardiovascular disease. It has been observed that whole blood, being a suspension of blood cells in plasma, behave as a non-Newtonian fluid. Several attempts [1-8] have been made to understand the flow characteristic of blood through arteries under various assumptions. But most of these studies have failed to estimate on an important role in blood rheology is the motion of erythrocytes, white blood cells and platelets in plasma.

Eringen [9, 10] introduced the theory of micropolar fluid, which exhibits microscopic effects arising from the local structure and microrotation of fluid microelements are considered to be rigid. A subclass of microfluids such as liquid crystals, suspensions and animal blood are treated as micropolar fluid, which takes into account the rotation of fluid particles by means of an independent kinematic vector called the microrotation vector. With this end in view, several investigators [12-14] considered micropolar fluid as a blood flowing through an arterial stenosis. They suggested that blood exhibits remarkable non-Newtonian properties, since blood is a suspension of neutrally bouyant deformable particles in viscous fluid. Devanathan and Parvathamma [15] developed a mathematical model for steady flow of micropolar fluid through an arterial stenosis and their results indicated that the axial velocity is highly affected owing to the presence of microelements in the fluid. However, all the above studies did not considered the effect of body acceleration. Although, under normal physiological conditions, flow properties of blood depends upon pumping action of the heart, which in turn produces a pulsatile pressure gradient through the cardiovascular system. Although, there may be some another important situation under which human beings experiences whole body acceleration. It may occur for a short or a long period of time. Such situations arise day to day life of human being e.g., driving a vehicle, flying in an aircraft, due to which there may occur, the serious health problems like headache, loss of vision, increase of pulse rate and hemarage in face, neck and eye socket. Keeping this in mind, several investigators [16-24] carried out their studies pertaining to blood flow through stenosed arteries with a periodic body acceleration. Chakravarty and Sannigrahi [25] studied theoretically the flow characteristics of blood through an artery in the presence of multi-stenosis when it is subjected to whole body acceleration. Wherein they considered a flexible cylindrical

tube containing a homogeneous Newtonian fluid with variable viscosity characterising blood. But all the above studies neither consider the effect of body acceleration on pulsating blood flow nor includes the effect of magnetic field when blood is considered to be micropolar fluid model. Since blood consists of suspension of red cells containing hemoglobin, which contains iron oxide, it is quite apparent that blood is electrically conducting and exhibits magnetohydrodynamic flow characteristics. If a magnetic field is applied to a moving and electrically conducting fluid, it will induce electric as well as magnetic fields. The interaction of these fields produce a body force known as Lorentz force, which has a tendency to slow down the fluid motion. Such analysis may be useful for pumping of blood and magnetic resonance imaging (MRI). The magnetic field can also be used for controlling blood flow during surgery. Many authors [26-29] have investigated the flow of blood through arteries in the presence of magnetic field under different situations. The steady MHD flow of a viscous fluid in a slowly varying channel in the presence of a uniform magnetic field was explored by Misra et al [30]. Recently Misra and Shit [31] investigated the flow and heat transfer of a viscoelastic electrically conducting fluid under the action of a magnetic field. They made an observation that the temperature of blood increases as the strength of magnetic field increases is likely to be useful in the development of new heating methods. With the above discussion in mind, a good attempt is made in the present theoretical investigation to examine the pulsatile blood flow characteristics through a stenosed artery experiencing periodic body acceleration by treating micropolar fluid model for blood in the presence of magnetic field. The motivation of this study is to analyze the effect of microrotation component and axial velocity when an external magnetic field and body accelerations are applied. In addition, in view of the information stated above, we have also performed a heat transfer phenomena of the problem in question.

2 Mathematical Formulation

Let us consider a fully developed unsteady, laminar, incompressible and axially symmetric two dimensional flow through a stenosed artery. An external magnetic field is applied to a pulsatile flow of blood, which is characterized by a micropolar fluid model under periodic body acceleration. Let (r^*, θ^*, z^*) be the co-ordinates of the material point in a cylindrical polar co-ordinate system, of which z^* is taken as central axis of the artery. We denote u^* and v^* the velocity

components of blood along the axial and radial directions respectively and w^* the component of microrotation. For axisymmetry, we assumed that the flow variation is independent of θ^* . Owning to the above considerations, the governing equations of the present problem may be written in the form,

$$\frac{\partial u^*}{\partial z^*} + \frac{v^*}{r^*} + \frac{\partial v^*}{\partial r^*} = 0 \quad (1)$$

$$\rho \left(\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial z^*} + v^* \frac{\partial u^*}{\partial r^*} \right) = -\frac{\partial p^*}{\partial z^*} + (\mu + k) \left(\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial u^*}{\partial r^*} + \frac{\partial^2 u^*}{\partial z^{*2}} \right) + k \left(\frac{\partial w^*}{\partial r^*} + \frac{w^*}{r^*} \right) - \sigma B_0^2 u^* + \rho G^*(t^*), \quad (2)$$

$$\rho \left(\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial z^*} + v^* \frac{\partial v^*}{\partial r^*} \right) = -\frac{\partial p^*}{\partial r^*} + (\mu + k) \left(\frac{\partial^2 v^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial v^*}{\partial r^*} - \frac{v^*}{r^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}} \right) - k \frac{\partial w^*}{\partial z^*}, \quad (3)$$

$$\rho j \left(\frac{\partial w^*}{\partial t^*} + u^* \frac{\partial w^*}{\partial z^*} + v^* \frac{\partial w^*}{\partial r^*} \right) = -k \left(2w^* + \frac{\partial u^*}{\partial r^*} - \frac{\partial v^*}{\partial z^*} \right) + \gamma \left(\frac{\partial^2 w^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial w^*}{\partial r^*} - \frac{w^*}{r^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}} \right), \quad (4)$$

$$\frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial z^*} + v^* \frac{\partial T^*}{\partial r^*} = \frac{\kappa_0}{\rho C_p} \left(\frac{\partial^2 T^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial T^*}{\partial r^*} + \frac{\partial^2 T^*}{\partial z^{*2}} \right) + \frac{\sigma B_0^2}{\rho C_p} u^{*2} \quad (5)$$

where ρ represents the blood density, p the blood pressure, μ the viscosity of blood, k the rotational viscosity, σ the electrical conductivity, B_0 the applied magnetic field intensity, γ the spin gradient viscosity is assumed to be $\gamma = j(\mu + k/2)$, j the micro-gyration constant, T^* is the temperature, C_p the specific heat and κ_0 denotes thermal conductivity. In the above, we have neglected the effect of induced magnetic field due to low magnetic Reynolds number ($R_m \ll 1$). For $t^* > 0$, the flow is assumed to have a periodic body acceleration $G^*(t^*)$ appeared in equation (2) has the expression of the form

$$G^*(t^*) = a^*(\cos \omega_b t^* + \phi_g) \quad (6)$$

where a^* denote the amplitude of body acceleration, ω_b the frequency of body acceleration and ϕ_g the phase difference.

Let us introduce the following non-dimensional variables to put the above equations in dimensionless form:

$$z = \frac{z^*}{R_0}, \quad r = \frac{r^*}{R_0}, \quad u = \frac{u^*}{\omega R_0}, \quad v = \frac{v^*}{\omega R_0}, \quad w = \frac{w^*}{\omega}, \quad p = \frac{p^*}{\mu \omega}, \quad t = \omega t^*, \quad J = \frac{j}{R_0^2}, \quad \theta = \frac{T^* - T_\infty}{T_w - T_\infty}.$$

Using these dimensionless variables the equations (1)-(5) becomes

$$\frac{\partial u}{\partial z} + \frac{v}{r} + \frac{\partial v}{\partial r} = 0 \quad (7)$$

$$\alpha^2 \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} + v \frac{\partial u}{\partial r} \right) = -\frac{\partial p}{\partial z} + (1+K) \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} \right) + K \left(\frac{\partial w}{\partial r} + \frac{w}{r} \right) - H^2 u + G(t) \quad (8)$$

$$\alpha^2 \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial z} + v \frac{\partial v}{\partial r} \right) = -\frac{\partial p}{\partial r} + (1+K) \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2} \right) - k \frac{\partial w}{\partial z} \quad (9)$$

$$\alpha^2 J \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial z} + v \frac{\partial w}{\partial r} \right) = -K \left(2w + \frac{\partial u}{\partial r} - \frac{\partial v}{\partial z} \right) + m \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \frac{w}{r^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (10)$$

$$\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial z} + v \frac{\partial \theta}{\partial r} = \frac{1}{\alpha^2 P_r} \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right) + \frac{E_c H^2}{\alpha^2} u^2 \quad (11)$$

where the non-dimensional parameters appeared in equations (8)-(11) are defined as the ratio of viscosity $K = \frac{k}{\mu}$, the material constant $m = \frac{\gamma}{\mu R_0^2}$, Womersley number $\alpha = R_0 \sqrt{\frac{\rho \omega}{\mu}}$, Hartmann number $H = R_0 B_0 \sqrt{\frac{\sigma}{\mu}}$, Prandtl number $P_r = \frac{\mu C_p}{\kappa_0}$, and Eckert number $E_c = \frac{\omega^2 R_0^2}{C_p (T_w - T_\infty)}$.

Using the non-dimensional quantities $a_0 = \frac{\rho R_0 a^*}{\mu \omega}$, $b = \frac{\omega b}{\omega}$, expression for the body acceleration takes the form

$$G(t) = a_0 \cos(bt + \phi_g) \quad \text{with } t \geq 0 \quad (12)$$

Also the non-dimensional form of pressure gradient and wall motion are assumed as

$$-\frac{\partial P(t)}{\partial z} = \bar{K} + K_p \cos(t) \quad (13)$$

and

$$R(t) = \bar{R} [1 + K_r \sin(t + \phi_r)] \quad (14)$$

where \bar{R} , K_r , ϕ_r , \bar{K} , and K_p stand for the mean radius, amplitude of arterial wall motion, the phase difference, constant amplitude of pressure gradient and amplitude of the pulsatile component gives rise to systolic and diastolic pressure. It is clear from equations (8)-(11) that

when K and m are set to zero, the system of equations reduces to a classical Newtonian fluid model.

Let us simplify the equations of motion by using long wavelength approximation ($\frac{R}{\lambda} \ll 1$), where the lumen of the arterial radius R is sufficiently smaller than the wavelength λ of pressure wave. Under this assumption the axial viscous transport terms are negligible from (8)-(11). Therefore the equation (8) reduces to simply $\frac{\partial p}{\partial r} = 0$ and others becomes

$$\frac{\partial u}{\partial z} + \frac{v}{r} + \frac{\partial v}{\partial r} = 0 \quad (15)$$

$$\alpha^2 \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} + v \frac{\partial u}{\partial r} \right) = -\frac{\partial p}{\partial z} + (1+K) \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + K \left(\frac{\partial w}{\partial r} + \frac{w}{r} \right) - H^2 u + G(t) \quad (16)$$

$$\alpha^2 J \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial z} + v \frac{\partial w}{\partial r} \right) = -K \left(2w + \frac{\partial u}{\partial r} - \frac{\partial v}{\partial z} \right) + m \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \frac{w}{r^2} \right) \quad (17)$$

$$\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial z} + v \frac{\partial \theta}{\partial r} = \frac{1}{\alpha^2 P_r} \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) + \frac{E_c H^2}{\alpha^2} u^2 \quad (18)$$

The boundary conditions for the present problem are described as follows:

Along the central line of the tube (at $r = 0$) the radial velocity, axial velocity gradient and spin of microrotation vanishes,

$$i.e., \text{ at } r = 0, \quad v = w = \frac{\partial u}{\partial r} = \frac{\partial \theta}{\partial r} = 0. \quad (19)$$

and at the wall

$$i.e., \text{ at } r = R(z, t), \quad u = w = 0 \quad \theta = 1.0 \quad \text{and} \quad v = \frac{\partial R}{\partial t}. \quad (20)$$

The geometry of the stenosis is described mathematically in non-dimensional form as (cf. Fig. 1)

$$\begin{aligned} R(z, t) &= \bar{R} \left[1 - \frac{\delta}{2R_0} \left\{ 1 + \cos \left(\frac{2\pi}{l_0} \left(z - d - \frac{l_0}{2} \right) \right) \right\} \right] (1 + K_r \sin(t + \phi_r)); \quad \text{when } d < z < d + l_0, \\ &= \bar{R} [1 + K_r \sin(t + \phi_r)]; \quad \text{elsewhere} \end{aligned} \quad (21)$$

where \bar{R} denotes the mean radius of the artery, δ the depth of the stenosis, R_0 is the radius of the artery at normal pathological state.

3 Method of Solution

Since the governing equations are couple and non-linear in nature, finding analytic solution is impossible. To simplify the governing equations, we introduced a radial coordinate transformation given by

$$\xi = \frac{r}{R(z, t)}.$$

Using this transformation, the equations (15)-(18) can be written in the following form

$$\frac{\partial u}{\partial z} - \frac{\xi}{R} \frac{\partial u}{\partial \xi} \frac{\partial R}{\partial z} + \frac{v}{\xi R} + \frac{1}{R} \frac{\partial v}{\partial \xi} = 0, \quad (22)$$

$$\begin{aligned} \frac{\partial u}{\partial t} = \frac{\frac{\partial u}{\partial \xi}}{R} \left[\xi \left(\frac{\partial R}{\partial t} + u \frac{\partial R}{\partial z} \right) - v \right] - u \frac{\partial u}{\partial z} + \frac{(1+K)}{\alpha^2 R^2} \left[\frac{\partial^2 u}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial u}{\partial \xi} \right] + \frac{K}{\alpha^2 R} \left(\frac{\partial w}{\partial \xi} + \frac{w}{\xi} \right) \\ + \frac{1}{\alpha^2} \left[G(t) - H^2 u - \frac{\partial p}{\partial z} \right], \end{aligned} \quad (23)$$

$$\begin{aligned} \frac{\partial w}{\partial t} = \frac{\frac{\partial w}{\partial \xi}}{R} \left[\xi \left(\frac{\partial R}{\partial t} + u \frac{\partial R}{\partial z} \right) - v \right] - u \frac{\partial w}{\partial z} - \frac{K}{\alpha^2 J} \left(2w - \frac{\partial v}{\partial z} \right) - \frac{K}{\alpha^2 J R} \left[\frac{\partial u}{\partial \xi} + \xi \frac{\partial v}{\partial \xi} \frac{\partial R}{\partial z} \right] \\ + \frac{m}{\alpha^2 J R^2} \left[\frac{\partial^2 w}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial w}{\partial \xi} - \frac{w}{\xi^2} \right], \end{aligned} \quad (24)$$

$$\frac{\partial \theta}{\partial t} = \frac{\frac{\partial \theta}{\partial \xi}}{R} \left[\xi \left(\frac{\partial R}{\partial t} + u \frac{\partial R}{\partial z} \right) - v \right] - u \frac{\partial \theta}{\partial z} + \frac{1}{\alpha^2 P_r R^2} \left[\frac{\partial^2 \theta}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial \theta}{\partial \xi} \right] + \frac{E_c H^2}{\alpha^2} u^2, \quad (25)$$

The boundary conditions (19) and (20) are also transformed to

$$v = w = \frac{\partial u}{\partial \xi} = \frac{\partial \theta}{\partial \xi} = 0 \quad \text{at} \quad \xi = 0. \quad (26)$$

and

$$u = w = 0, \quad v = \frac{\partial R}{\partial t} \quad \text{and} \quad \theta = 1.0 \quad \text{at} \quad \xi = 1. \quad (27)$$

Let us multiply the equation (22) by ξR and integrating with respect to ξ between the limits 0 and ξ , we get

$$v(\xi, z, t) = \xi u \frac{\partial R}{\partial z} - \frac{R}{\xi} \int_0^\xi \xi \frac{\partial u}{\partial z} d\xi - \frac{2}{\xi} \frac{\partial R}{\partial z} \int_0^\xi \xi u d\xi. \quad (28)$$

Using the boundary condition (27) at $\xi = 1$ the equation (28) yeilds

$$\frac{1}{R} \frac{\partial R}{\partial t} = -\frac{2}{R} \frac{\partial R}{\partial z} \int_0^1 \xi u d\xi - \int_0^1 \xi \frac{\partial u}{\partial z} d\xi$$

and has the form

$$\int_0^1 \xi \frac{\partial u}{\partial z} d\xi = \int_0^1 \xi \left[\frac{1}{R} \frac{\partial R}{\partial t} f(\xi) - \frac{2}{R} \frac{\partial R}{\partial z} u \right] d\xi. \quad (29)$$

Let us choose $f(\xi) = -4(\xi^2 - 1)$ such that $\int_0^1 \xi f(\xi) d\xi = 1$.

Now equating the integrands from both sides of (29) we have

$$\frac{\partial u}{\partial z} = \frac{4(\xi^2 - 1)}{R} \frac{\partial R}{\partial t} - \frac{2}{R} \frac{\partial R}{\partial z} u, \quad (30)$$

which on substitution in (28), we obtain the radial velocity in the following form

$$v(\xi, z, t) = \xi \left[u \frac{\partial R}{\partial z} + (2 - \xi^2) \frac{\partial R}{\partial t} \right]. \quad (31)$$

In the next section, we present computational scheme for computing numerical solution of the velocity components.

4 Computational Scheme

Since our Physical domain is highly complex to reduce it to a simple rectangular domain, we used the co-ordinate transformation, presented in the previous section. To solve the equations (22)-(25) and (30) using a finite difference technique, we subdivided the rectangular domain into a network by drawing straight lines parallel to the co-ordinate axes. The solutions $u(z, \xi, t)$, $v(z, \xi, t)$, $w(z, \xi, t)$ and $\theta(z, \xi, t)$ at any mesh point in a computational domain are denoted by $u_{i,j}^k$, $v_{i,j}^k$, $w_{i,j}^k$ and $\theta_{i,j}^k$ in which

$$z_i = i\Delta z ; \quad i = 0, 1, \dots, M;$$

$$\xi_j = j\Delta \xi ; \quad j = 0, 1, \dots, N;$$

$$t_k = (k - 1)\Delta t ; \quad k = 1, 2, 3, \dots$$

where M and N are the maximum number of mesh points in the z and ξ -directions respectively.

Let $V_{i,j}^k$ stand for the variables u , v , w and θ , then the spatial and time derivatives are replaced by the central difference as

$(\frac{\partial V}{\partial \xi})_{i,j}^k = \frac{V_{i,j+1}^k - V_{i,j-1}^k}{2\Delta\xi}$, $(\frac{\partial V}{\partial z})_{i,j}^k = \frac{V_{i+1,j}^k - V_{i-1,j}^k}{2\Delta z}$, $(\frac{\partial^2 V}{\partial \xi^2})_{i,j}^k = \frac{V_{i,j+1}^k - 2V_{i,j}^k + V_{i,j-1}^k}{(\Delta\xi)^2}$, $(\frac{\partial V}{\partial t})_{i,j}^k = \frac{V_{i,j}^{k+1} - V_{i,j}^k}{\Delta t}$ and so on.

Using these finite differences, the discretized equations for (23)-(25) and (30) becomes

$$u_{i,j}^{k+1} = u_{i,j}^k + \Delta t \left[\frac{u_{i,j+1}^k - u_{i,j-1}^k}{2\Delta\xi R_i^k} \left(\xi_j \left(\frac{\partial R^k}{\partial t}_i + u_{i,j}^k \frac{\partial R^k}{\partial z}_i \right) - v_{i,j}^k \right) - u_{i,j}^k \frac{u_{i+1,j}^k - u_{i-1,j}^k}{2\Delta z} + \frac{1+K}{\alpha^2 (R_i^k)^2} \left(\frac{u_{i,j+1}^k - 2u_{i,j}^k + u_{i,j-1}^k}{(\Delta\xi)^2} + \frac{1}{\xi_j} \frac{u_{i,j+1}^k - u_{i,j-1}^k}{2\Delta\xi} \right) + \frac{K}{\alpha^2 R_i^k} \left(\frac{w_{i,j+1}^k - w_{i,j-1}^k}{2\Delta\xi} + \frac{w_{i,j}^k}{\xi_j} \right) + \frac{1}{\alpha^2} \left(G(t) - \frac{\partial p^k}{\partial z}_i - H^2 u_{i,j}^k \right) \right], \quad (32)$$

$$w_{i,j}^{k+1} = w_{i,j}^k + \Delta t \left[\frac{w_{i,j+1}^k - w_{i,j-1}^k}{2\Delta\xi R_i^k} \left(\xi_j \left(\frac{\partial R^k}{\partial t}_i + u_{i,j}^k \frac{\partial R^k}{\partial z}_i \right) - v_{i,j}^k \right) - u_{i,j}^k \frac{w_{i+1,j}^k - w_{i-1,j}^k}{2\Delta z} - \frac{K}{\alpha^2 J} \left(2w_{i,j}^k - \frac{v_{i+1,j}^k - v_{i-1,j}^k}{2\Delta z} \right) - \frac{K}{\alpha^2 R_i^k J} \left(\frac{u_{i,j+1}^k - u_{i,j-1}^k}{2\Delta\xi} + \xi_j \frac{\partial R^k}{\partial z}_i \frac{v_{i,j+1}^k - v_{i,j-1}^k}{2\Delta\xi} \right) + \frac{m}{\alpha^2 (R_i^k)^2 J} \left(\frac{w_{i,j+1}^k - 2w_{i,j}^k + w_{i,j-1}^k}{(\Delta\xi)^2} + \frac{1}{\xi_j} \frac{w_{i,j+1}^k - w_{i,j-1}^k}{2\Delta\xi} - \frac{w_{i,j}^k}{\xi_j^2} \right) \right], \quad (33)$$

$$\theta_{i,j}^{k+1} = \theta_{i,j}^k + \Delta t \left[\frac{\theta_{i,j+1}^k - \theta_{i,j-1}^k}{2\Delta\xi R_i^k} \left(\xi_j \left(\frac{\partial R^k}{\partial t}_i + u_{i,j}^k \frac{\partial R^k}{\partial z}_i \right) - v_{i,j}^k \right) - u_{i,j}^k \frac{\theta_{i+1,j}^k - \theta_{i-1,j}^k}{2\Delta z} + \frac{1}{\alpha^2 P_r (R_i^k)^2} \left(\frac{\theta_{i,j+1}^k - 2\theta_{i,j}^k + \theta_{i,j-1}^k}{(\Delta\xi)^2} + \frac{1}{\xi_j} \frac{\theta_{i,j+1}^k - \theta_{i,j-1}^k}{2\Delta\xi} \right) + \frac{E_c H^2}{\alpha^2} (u_{i,j}^k)^2 \right], \quad (34)$$

$$v_{i,j}^{k+1} = \xi_j \left[u_{i,j}^k \frac{\partial R^k}{\partial z}_i + (2 - \xi_j^2) \frac{\partial R^k}{\partial t}_i \right] \quad (35)$$

where $(\frac{\partial R}{\partial z})_i^k$ and $(\frac{\partial R}{\partial t})_i^k$ are computed at the arterial wall.

After having determined the values of the axial velocity, radial velocity, microrotation component and temperature one can easily obtain the fluid acceleration F , the volumetric flow rate Q , flow resistance λ , wall shear stress τ_w and the Nusselt number Nu from the following relations,

$$F_{i,j}^{k+1} = \frac{u_{i,j}^{k+1} - u_{i,j}^k}{\Delta t} + u_{i,j}^k \frac{u_{i+1,j}^k - u_{i-1,j}^k}{2\Delta z} - \frac{\xi_j}{R_i^k} \left(\frac{\partial R^k}{\partial t}_i + u_{i,j}^k \frac{\partial R^k}{\partial z}_i \right) \frac{u_{i,j+1}^k - u_{i,j-1}^k}{2\Delta\xi}$$

$$Q_i = 2\pi (R_i^k)^2 \int_0^1 \xi_j u_{i,j}^k d\xi_j$$

$$\lambda_i^k = \frac{|L \frac{\partial p^k}{\partial z}_i|}{Q_i^k},$$

$$Nu_i = \frac{1}{R_0^2 R_i^2} \left[\frac{\theta_{i,N}^k - \theta_{i,N-1}^k}{\Delta\xi} \right]$$

5 Computational Results and Discussion

The objective of the present study has been to investigate the flow characteristics of blood through stenosed artery under the action of a periodic body acceleration as well as an external magnetic field on the axial velocity by taking into account the effect of microrotation of blood cells. A specific numerical illustration has been taken to examining the applicability of the physiological data available in the existing scientific literatures [3,4,19-21,28,29]. The computational work has been carried out by using the following data:

$L = 5.0$, $d = 2.0$, $l_0 = 1.0$, $\delta = 0.1, 0.25, 0.50$; $a_0 = 0.0, 1.0, 2.0, 3.0$; $b = 1.0$, $\phi_g = 0.0$, $\phi_r = 0.0$, $R_0 = 1.0$, $\bar{R} = 1.0$, $K_p = 1.46$, $\bar{K} = 7.30$, $K_r = 0.05$, $f_p = 1.2$, $K = 0.0, 0.1, 0.2, 0.3$; $J = 0.1$, $m = 0.1, 0.01, 0.001$; $\alpha = 3.0$, $H = 0.0, 1.0, 2.0, 3.0, 4.0$; $E_c = 0.0002$,

and for a human body temperature, $T = 310K$ the value of $P_r = 21$ is considered for blood. For the sake of comparison, we have also examined the cases where $P_r = 7, 14$. We used $\Delta\xi = 0.025$, $\Delta z = 0.05$, $\Delta t = 0.001$ through out the computation. We observed that further reduction in $\Delta\xi$ and Δz does not bring about any significant change, which leads to the stability of the numerical methods.

Figs. 2(a)-(c) give the variation of axial velocity u along the radial direction ξ . In Fig. 2(a) dotted lines indicate the axial velocity distribution for Newtonian fluid model and solid lines represent that of the micropolar fluid model. From this figure, we observe that the axial velocity decreases with the increase of the Hartmann number H . The axial velocity occurs maximum at the central line of the artery in all four cases. One can note from Fig 2(a) that for a sufficient strength of magnetic field, the profiles for both the Newtonian and non-Newtonian models are same. The axial velocity is found to be greater in the case of Newtonian model than that of the micropolar fluid model. This happens due to the presence of microrotation of blood cells, creates an additional viscosity called rotational viscosity, which in turn diminishes axial velocity. Fig. 2(b) depicts the variation of axial velocity for different values of K , the ratio of viscosity $\frac{k}{\mu}$. When the values of K increases, the axial velocity decreases. The results, thus obtained due to the increase of the microrotations of blood cells in the lumen of the artery. The axial velocity is minimum in the vicinity of the arterial walls, which gives rise to conclude that the microrotation of blood cells occur in the plasma layer. The variation of axial velocity for different amplitudes of body acceleration is shown in Fig. 2(c). It reveals that the axial velocity increases as the am-

plitude of body acceleration increases. Thus in the presence of vibration environmental system may produce the increase in blood velocity.

Figs. 3(a)-(c) illustrate the distribution of micro-component of velocity w along the radial direction of the artery. Fig. 3(a) shows that the microrotation component increases with the increase of the ratio of the viscosity K , which in turn produces decrease of axial velocity. It has been shown in Fig. 3(b) that the microcomponent of velocity w increases with the decrease in micropolar material parameter m . The result lies in the fact that, when m decreases, the fluid viscosity μ increases, thereby the microcomponent of velocity increases. One can note that the effect of microrotation observed significantly near the arterial wall. The variation of microrotation component is found to increase with the increase of the blockage of an artery presented in Fig. 3(c). In all these cases $\delta = 0.1, 0.25$, and 0.5 have been considered for computation and which are respectively represent the 10%, 25% and 50% of the blockage of the arterial segment.

In Figs. 4(a) and (b) we present the variation of dimensionless temperature distribution at the throat of the stenosis and the rate of heat transfer at the arterial wall along with the axial direction. It is observed from Fig. 4(a) that the temperature decreases with the increase of the Prandtl number P_r . However, the temperature increases with the increase of Hartmann number H . It is interesting to mention here that the temperature rapidly increases with the increase of H up to 4.0 (T) and beyond which, no significant change is noticed. The rate of heat transfer (Nu) increases with the increase of prandtl number P_r , shown in Fig. 4(b). The rate of heat transfer is maximum at the throat of the stenosis and is minimum at the down stream of the stenosis than that of upstream. Thus the temperature can be increased whenever necessary by the application of the certain magnetic field strength. It is well known that the objective of the hyperthermia in cancer therapy is to raise the temperature above a therapeutic value 42°C , while maintaining the surrounding temperature at sublethal temperature [32].

The important characteristics of blood flow are the wall shear stress τ_w and the volumetric flow rate Q . Figs. 5(a)-(d) give the variation of wall shear stress τ_w with axial distance z and Time $T = t/2\pi f_p$ for different height of the stenosis δ , different values of Hartmann number H , amplitude of body acceleration a_0 and the material parameter K . In Fig. 5(a), we observe that

the wall shear stress increases as the height of the stenosis increases. It is interesting to note from this figure that the wall shear stress significantly changes at the down stream of the stenosis, whereas, at the upstream of the stenosis no change is occur. Fig. 5(b) presents the variation of wall shear stress τ_w with time T for different values of the Hartmann number H . It shows that the wall shear stress varies periodically with time T and the peak value of each oscillation decreases with the increase of H . Fig. 5(c) shows that the amplitude of the oscillation of wall shear stress increases with the increase of the amplitude of the body acceleration. However, there is no significant change on the wall shear stress for the variation of material parameter K (cf. Fig. 5(d)). This is due to the fact that the microelements are unable to rotate close to the arterial wall. Although the peak value of oscillation increases with the increase of K , but the change is insignificant.

The variation of volumetric flow rate Q with time T presented through the Figs. 6(a)-(c). We observe that the flow rate Q decreases with the increase of Hartmann number H as well as the increase of the material parameter K , while the trend is reversed in the case of the amplitude of body acceleration $G(t)$. In all these three cases, the flow rate Q varies periodically with time T . It thus turns out that under the action of a magnetic field, volume of blood flow can be controlled during surgeries. The volumetric flow has an enhancing effect with its peak value of oscillation on the vibration environmental system.

Figs. 7(a) and (b) depict the variation of fluid acceleration F along the radial coordinate ξ for different values of the amplitude of body acceleration and different magnetic field intensity given by H . For each of these cases, the fluid acceleration F is maximum at the central line and then it is gradually diminishes and ultimately vanishes at the wall of the artery. It is also observe from Fig. 7(a) that the magnitude of the fluid acceleration F increases as the amplitude of body acceleration increases. Thus in the presence of an environmental system fluid acceleration enhances. However, the magnitude of the fluid acceleration F decreases with the increase of the Hartmann number H , shown in Fig. 7(b). One can mention here that the rate of increase or decrease of fluid acceleration F significantly affected by the application of magnetic field more than in the presence of body acceleration.

The variation of flow resistance λ along with the height of the stenosis δ has been shown in Figs. 8(a)-(c). It has been observed from Fig. 8(a) that the resistance to flow decreases with an increase in amplitude a_0 of the body acceleration. On the other hand flow resistance increases with the increase of the values of H and K , shown in Figs. 8(b) and (c). It is worth mentioning that in all these figures, the flow resistance λ gradually increases as the depth of the stenosis increases. It is learn from these figures that the flow resistance remains fixed upto a certain height of the stenosis that is upto 25% of the blockage of an artery (called mild stenosis) beyond which it increases significantly.

6 Concluding Remarks

The present mathematical analysis is motivated towards the modelling of pulsatile blood flow through a stenosed artery under the combined influence of an external magnetic field and periodic body acceleration. The mathematical model is developed towards the application in blood flow in order to consider the effect of micro-rotation of micro-particles suspended in plasma. When a magnetic field is applied to the blood flow, the erythrocytes orient their disk plane parallel to the axis of magnetic field, creates an additional viscosity, which in turn produces decrease of blood velocity. This interesting observation can be seen in Fig. 2(b). The temperature of blood can be increased by the application of magnetic field. It is also concluded that after certain magnetic field strength, the temperature is constant. This important result can be useful in the therapeutic treatment of patient [32]. The present study also enables to have an estimate of the effects of microcomponent of velocity under the influence of periodic body acceleration. However, the consideration of pulsatile flow on the cases of the severe stenosis are the further scope of the study.

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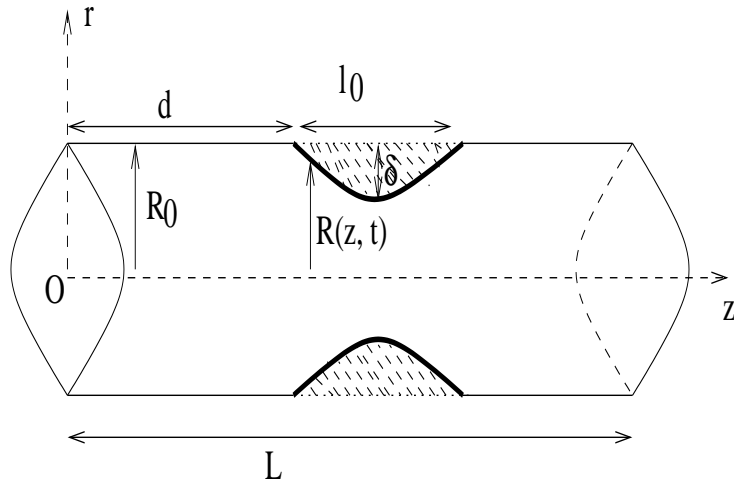


Fig. 1 A physical sketch of the model artery with stenosis.

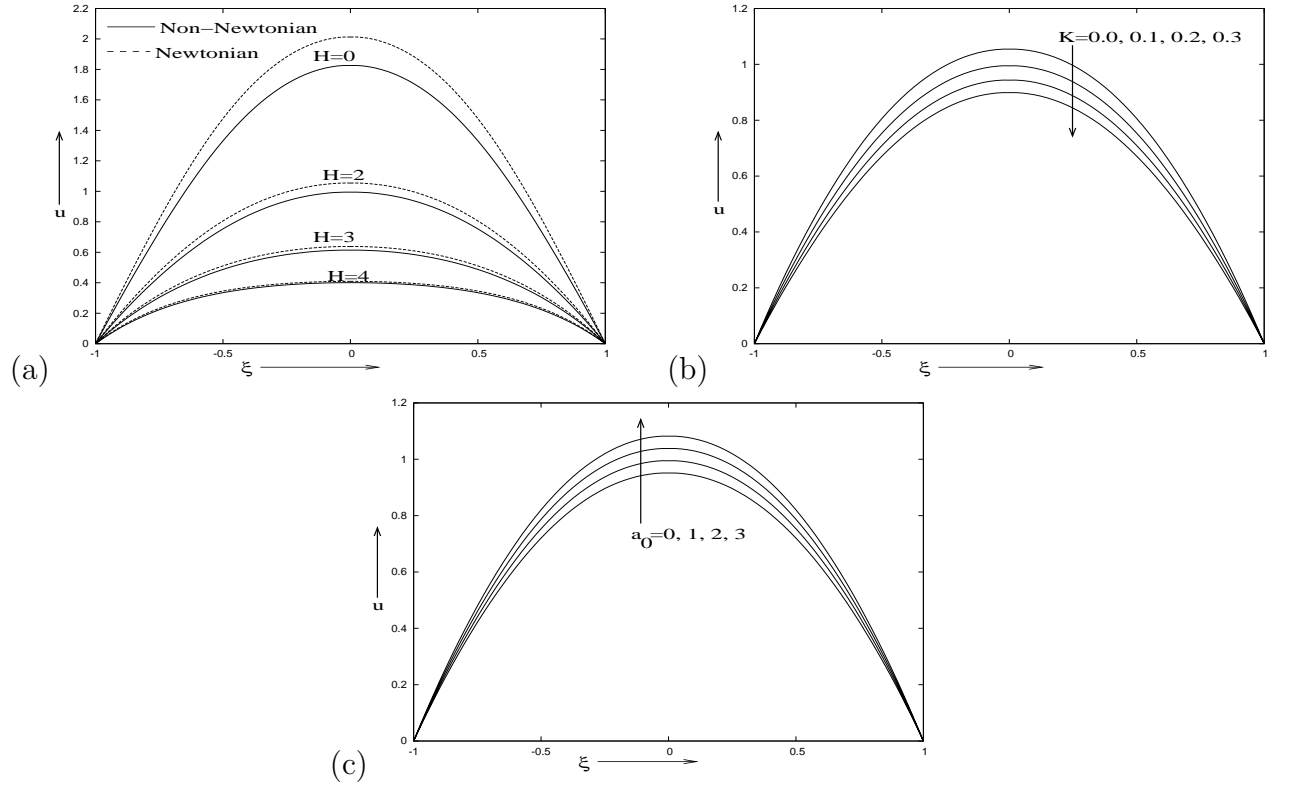


Fig. 2 Variation of axial velocity u at the throat of the stenosis for (a) different values of the Hartmann number H , when $K = 0.1$, $a_0 = 1.0$; (b) different values of K when $H = 2.0$, $a_0 = 1.0$; (c) different amplitude of body acceleration a_0 when $H = 2.0$ and $K = 0.1$.

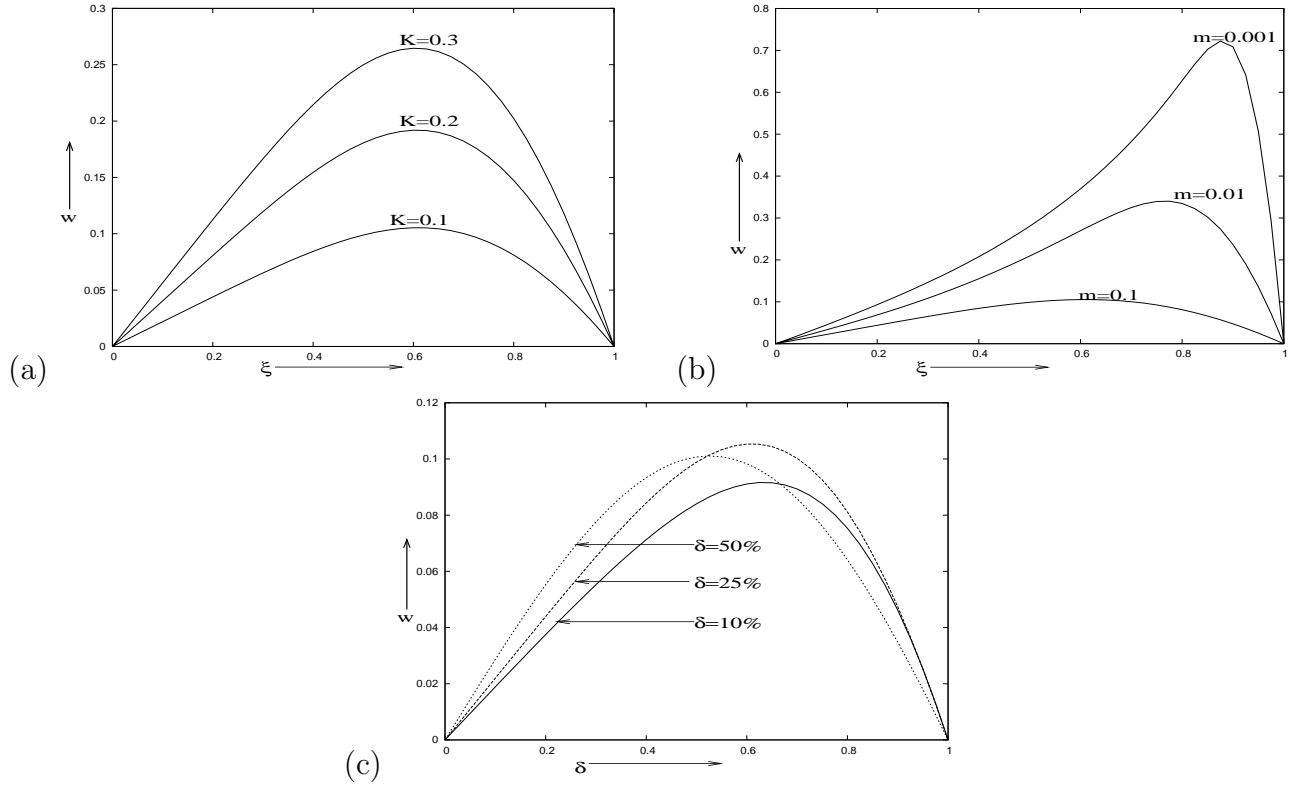


Fig. 3 Variation of microrotation component w at the throat of the stenosis (a) for different values of material parameter K with $m = 0.1, \delta = 0.25$; (b) for different values of m , with $K = 0.1, \delta = 0.25$; (c) for different values of δ , with $K = 0.1, m = 0.1$.

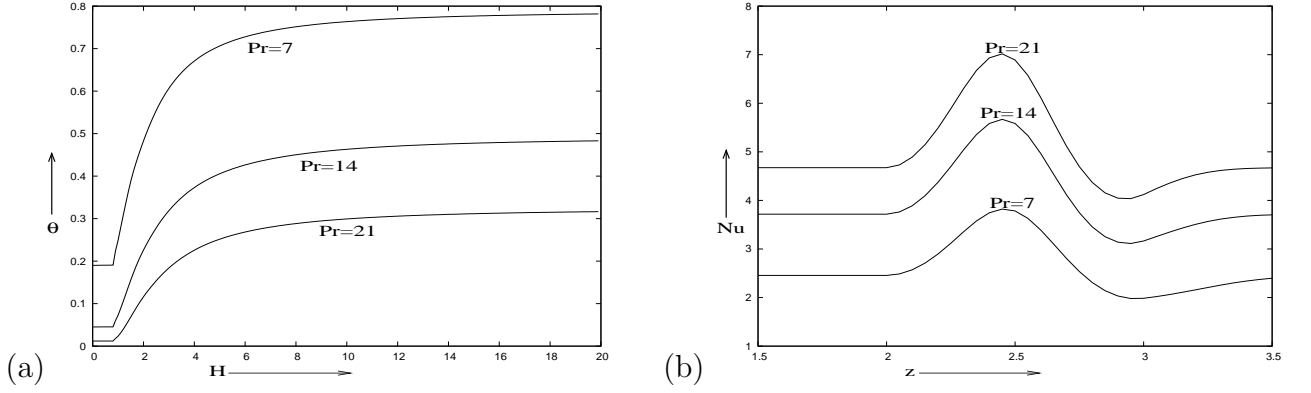


Fig. 4 (a) Distribution of temperature θ for different values of H and Pr . (b) Variation of Nusselt number Nu along the axis of the artery for different values of Prandlt number Pr . With $a_0 = 1.0$, $b = 1.0$, $\delta = 0.25$, $\phi_g = 0.0$, $\phi_r = 0.0$, $K = 0.1$, $m = 0.1$, $\alpha = 3.0$, $H = 2.0$,

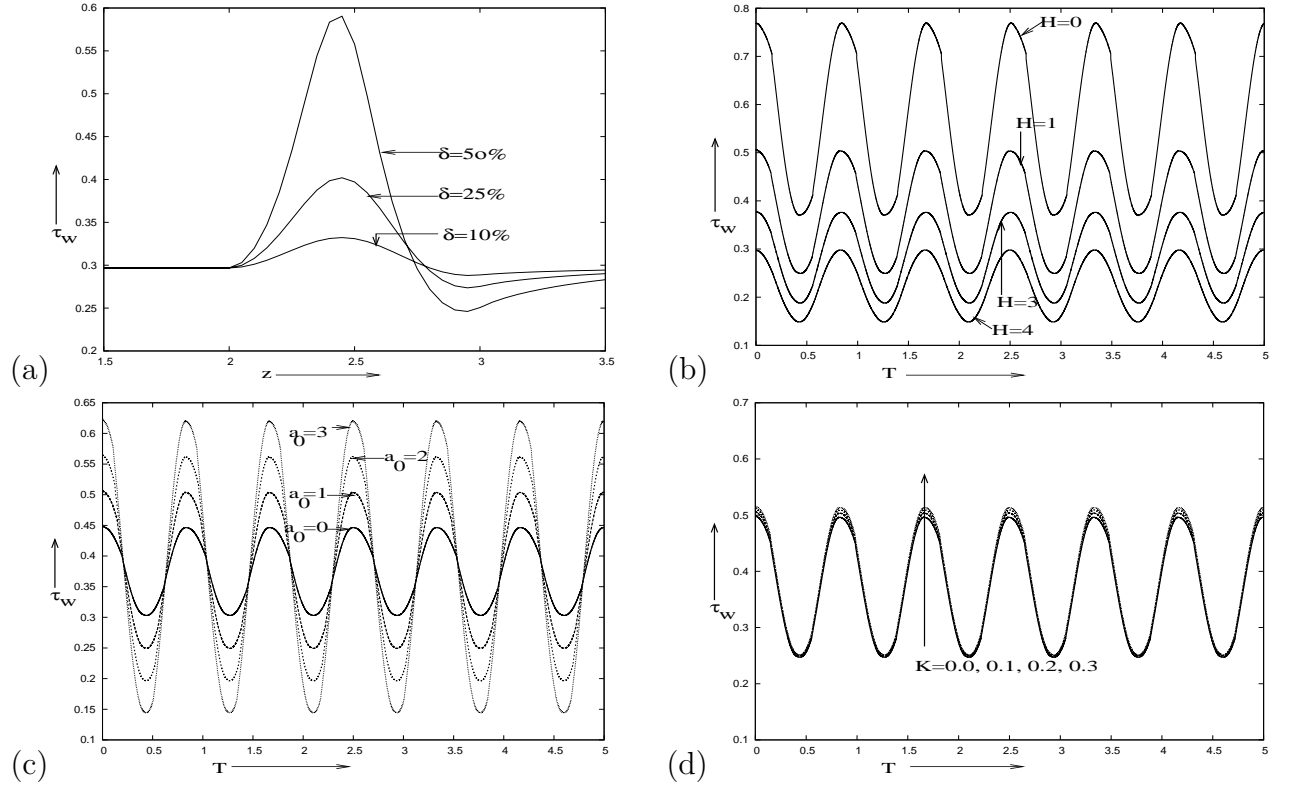
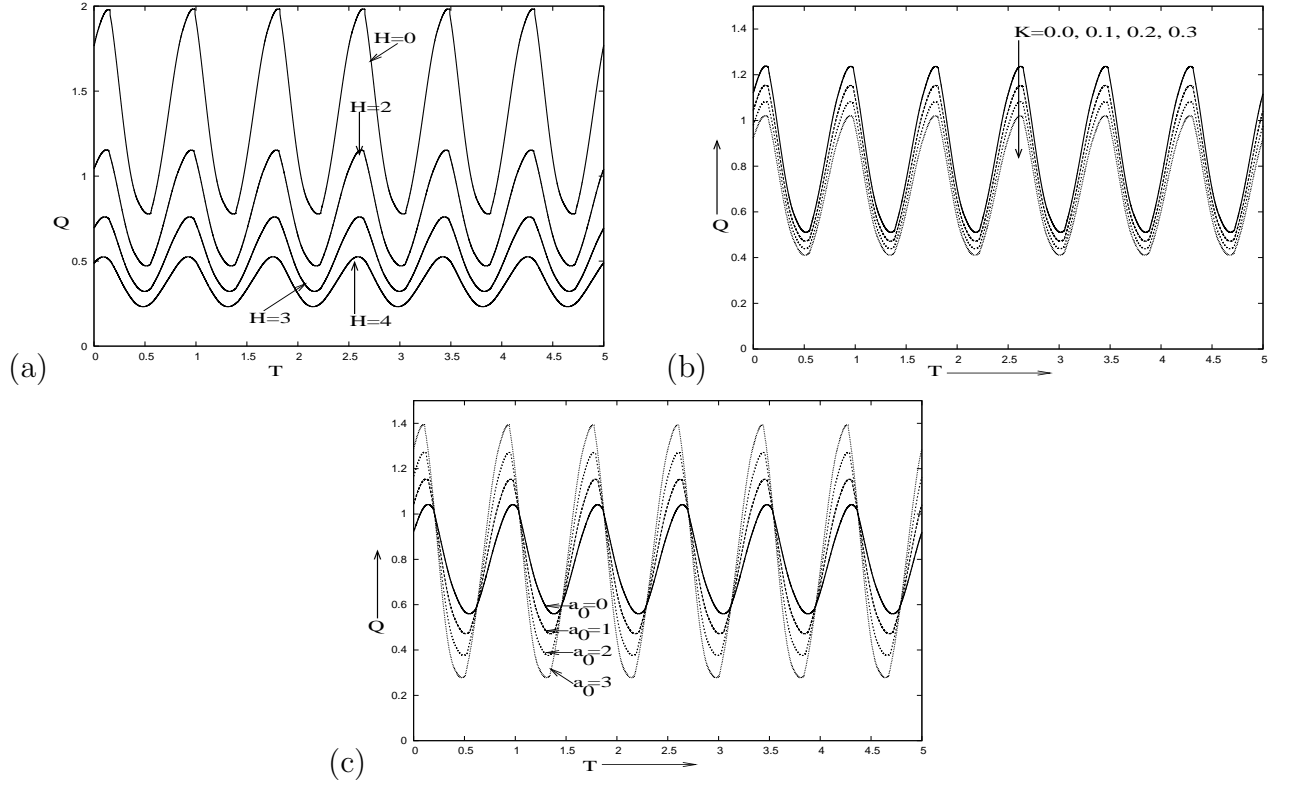


Fig. 5 Variation of wall shear stress τ_w (a) along the axis of the artery for different height of the stenosis δ , with $H = 2.0$, $a_0 = 1.0$, $K = 0.1$; (b) with time T for different values of H , when $\delta = 0.25$, $a_0 = 1.0$, $K = 0.1$; (c) with time T for different amplitude of body acceleration a_0 , when $\delta = 0.25$, $H = 2.0$, $K = 0.1$; (d) with time T for different values of K , when $\delta = 0.25$, $a_0 = 1.0$, $H = 2.0$.



Figs. 6 Variation of volumetric flow rate Q with time T (a) for different values of H , when $K = 0.1$, $a_0 = 1.0$; (b) for different values of H , when $a_0 = 1.0$, $H = 2.0$ and (c) for different values of a_0 , when $K = 0.1$, $H = 2.0$.

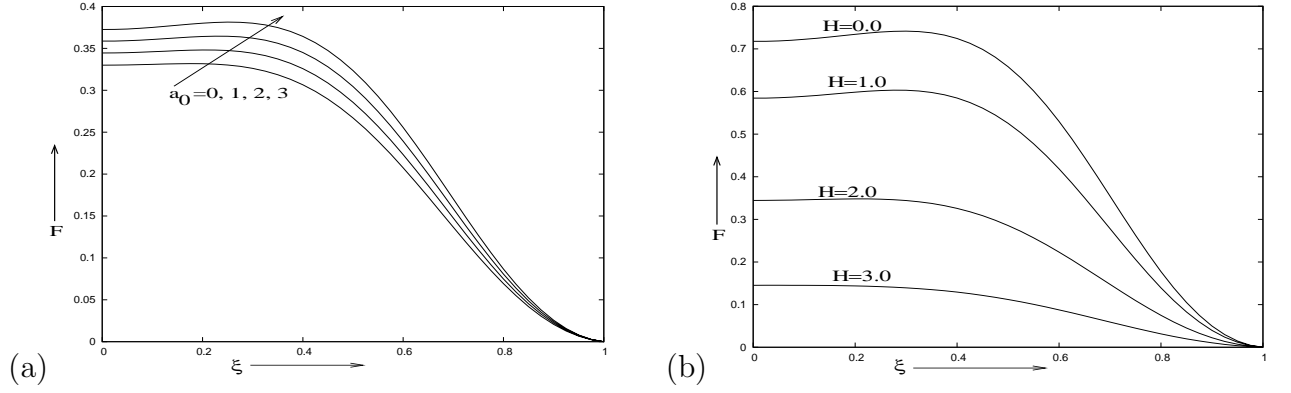


Fig. 7 Variation of fluid acceleration F along the radial direction for (a) different amplitude of body acceleration a_0 when $H = 2.0$ and (b) for different values of Hartmann number H when $a_0 = 1.0$.

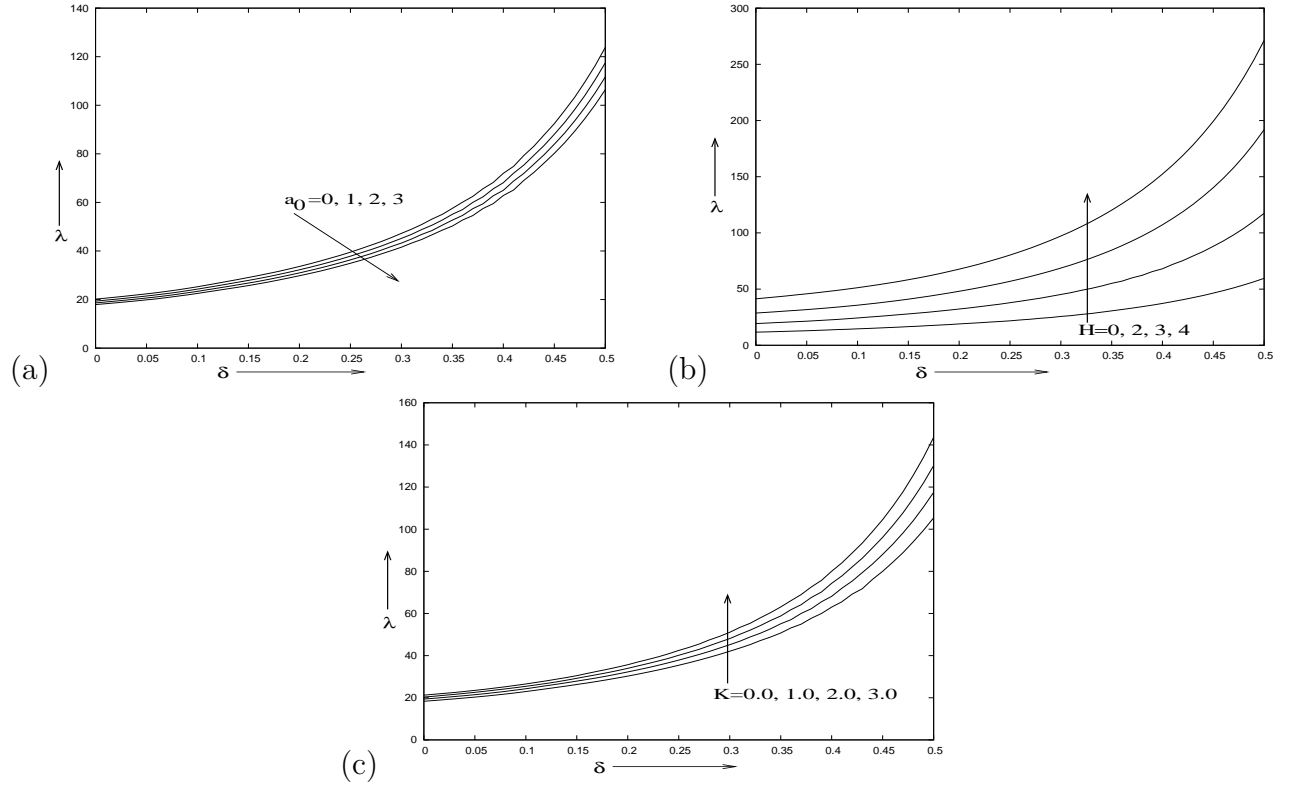


Fig. 8 Variation of flow resistance λ with depth of the stenosis (a) for different values of a_0 , when $H = 2.0$, $K = 0.1$, (b) for different values of Hartmann number H , when $a_0 = 1.0$, $K = 0.1$ and (c) for different values of K , when $H = 2.0$, $a_0 = 1.0$.